More Powerful Unit Root Tests with Non-normal Errors

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Abstract

This paper proposes new unit root tests that are more powerful when the error term follows a non-normal distribution. The improved power is gained by utilizing the additional moment conditions embodied in non-normal errors. Specifically, we follow the work of Im and Schmidt (2008), using the framework of generalized methods of moments (GMM), and adopt a simple two-step procedure based on the "residual augmented least squares" (RALS) methodology. Our RALS-based unit root tests make use of non-linear moment conditions through a computationally simple procedure. Our Monte Carlo simulation results show that the RALS-based unit root tests have good size and power properties, and they show significant efficiency gains when utilizing the additional information contained in non-normal errors—information that is ignored in traditional unit root tests.

JEL Classification: C22, C12, C13.

Key Words: Unit root test, Generalized methods of moments (GMM), Residual augmented least squares (RALS), Non-normality.

1 Introduction

In this paper, we suggest new unit root tests that utilize the information contained in non-normal errors in order to achieve improved power. The search for a more powerful unit root test is not a trivial concern since it is well known that the traditional tests have relatively low power. Our approach re-examines the conventional wisdom which tends to ignore or downplay the non-normal distribution of the error term in least squares estimation. While it is true that the limiting distribution of the usual unit root tests using least squares estimation is not affected by non-normality of the errors, this result does not necessarily imply that the information embodied in non-normal errors is useless. This paper shows that, in the presence of non-normality, more powerful unit root tests can be obtained by utilizing the information in the higher moments of the errors—moments that are not used in the case of traditional unit root tests estimated by least squares. To achieve the efficiency gains and improved power when constructing our tests, we extend the work of Im and Schmidt (2008) and adopt a simple two-step procedure following the "residual augmented least squares" (RALS) estimation. Although Im and Schmidt (2008) consider a family of models where the estimator is consistent at rate $\sqrt{T}$, we show that the same efficiency gains will follow for the estimator having a higher rate of convergence when dealing with non-stationary data. We first consider GMM estimators utilizing non-linear moment conditions to test for a unit root, and show that the linearized RALS estimators using the same moment conditions are asymptotically equivalent to the GMM estimators. Then, we will show that the newly suggested RALS-based unit root tests utilizing higher moment conditions show substantial power gains when the errors are non-normal.

When dealing with real-world data, it is not uncommon to find non-normality in many time series variables. Non-normal distributions can occur for a variety of reasons, and this phenomenon may not easily be distinguished from some forms of non-linearity. For example, many financial time series variables have fat-tailed or leptokurtic distributions, which often are modeled in a non-linear framework. In addition, some financial variables are characterized by skewed distributions, which can occur when an asymmetric relationship exists in the data. In such instances, the non-linear exponential smooth transition autoregressive (ESTAR) or logistic smooth transition autoregressive (LSTAR) models often are applied. Furthermore, some economic time series variables have a mixture of different distributions, which typically would be modeled in a non-linear framework including regime switching models. Clearly, these examples illustrate that many cases of non-normality may be attributed to non-linearity. Since traditional unit root tests ignore the information contained in non-normal errors, it would be prudent to explore the way to achieve improved power. RALS-based unit root tests provide a convenient procedure with which we can utilize the higher moment conditions that exist under non-normality.

A handful of authors previously have investigated the possibility of utilizing the information contained in non-normal errors when testing for a unit root. For example, Cox and Llatas (1991) studied the asymptotic distribution of maximum likelihood estimators (MLE) in the Dickey-Fuller regression assuming that the true error density is known. Lucas (1995) derived the asymptotic distribution of the unit root test statistic based on the M-estimator. Shin and So (1999) considered adaptive maximum likelihood estimators. The motivation of our RALS-based unit root tests can be understood in a similar context. Our tests differ from the above-mentioned approaches in a few important regards. First, it is not necessary for us to specify a particular density function for the error term, the score function, or a specific non-linear functional form. Since these generally are unknown a priori, our approach offers a great advantage over the aforementioned tests. Second, we do not need a non-linear optimization procedure or convergence of iterations. Thus, a second major advantage of our approach is the use of a linear framework that relies on least squares estimation. The salient feature of our RALS-based unit root tests is that they make use of non-linear moment conditions through a computationally simple procedure. In addition, we note that the distribution of the RALS-based estimator is asymptotically identical to that of the GMM estimator using moment conditions. Thus, the linearized RALS unit root tests can achieve the same efficiency gains as that of the GMM estimator using non-linear moment conditions.

Our RALS-based tests are closely related to the pioneering work of Hansen (1995) who suggested augmenting the unit root testing equation with stationary covariates, if available, to gain increased power. Wooldridge (1993) and Qian and Schmidt (1999) also noted that it is possible to increase efficiency of estimation by augmenting the testing equation with variables that are correlated with the error term. In so doing, the error variance of the regression augmented with the stationary covariates will be smaller than that of the usual Dickey-Fuller regression. Hansen's test requires other stationary covariate variables to be added to the testing equation. They must be correlated with the error term but uncorrelated with the regressors. It is ofen difficult to find such variables. The RALS unit root tests are useful in this regard. Instead of looking for other stationary covariates, we can use the information that a time series itself contains. Hansen (1995) shows that the asymptotic distribution of the unit root test using stationary covariates is a mixture of the Dickey-Fuller distribution and the standard normal distribution. Our RALS-based test statistic has the same asymptotic distribution.

The rest of the paper is organized as follows. In Section 2, we derive the asymptotic distribution of the GMM-based unit root tests. In Section 3, we propose the RALS-based unit root tests and provide the asymptotic distribution when the errors are non-normal. In Section 4, we provide simulation results to examine the performance of the RALS-based unit root tests and compare them with other tests. Section 5 provides an empirical example and section 6 provides concluding remarks.

2 GMM Unit Root Test Statistics

We first consider GMM estimators utilizing some moment restrictions for unit root tests and derive the asymptotic distributions of the GMM estimators, as well as their associated t-statistics. Consider a time series that follows:

$$y_t = \phi y_{t-1} + \varepsilon_t,\ t = 1, 2, .., T, \tag{1}$$

where $\{\varepsilon_t\}_{t=1}^{\infty}$ is a sequence of innovations. For the unit root hypothesis, we are interested in testing $H_0 : \phi = 1$ against the alternative hypothesis $H_A : \phi < 1$. We assume:

Assumption 1. $\varepsilon_t = \sum_{j=1}^{p} a_j \varepsilon_{t-j} + e_t$, $t = 1, 2, .., T$, where $\{e_t\}_{t=1}^{\infty}$ is an iid sequence with zero mean and a finite second moment σ_e^2, and all roots of $a(z) = 1 - \sum_{j=1}^{p} a_j z^j$ lie outside of the unit circle.

When Assumption 1 is met, one may consider the Dickey-Fuller testing equation:

$$\Delta y_t = \beta y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + e_t, \; t = 1, 2, .., T, \tag{2}$$

where $\Delta y_t = y_t - y_{t-1}$. Let $\hat{\beta}_{LS}$ be the least squares estimator of β in regression (2). We denote t_{LS} as its t-statistic. Then it is well known, under the null hypothesis, that:

$$T\hat{\beta}_{LS} \Rightarrow a(1) \left(\int_0^1 W(r)^2 dr \right)^{-1} \int_0^1 W(r) dW(r), \tag{3}$$

and:

$$t_{LS} \Rightarrow \left(\int_0^1 W(r)^2 dr \right)^{-1/2} \int_0^1 W(r) dW(r) = DF, \tag{4}$$

where $a(1) = 1 - \sum_{j=1}^{p} a_j$, and $W(r)$ is the standard Brownian motion on $r \in [0, 1]$.

Let $\xi_t = (\Delta y_{t-1}, \Delta y_{t-2}, ..., \Delta y_{t-p})'$, and $z_t = (y_{t-1}, \xi_t')'$. Suppose we have $J \times (p + 1)$ additional moment conditions:

$$E\left[g(e_t) \otimes z_t\right] = 0, \; t = 1, 2, ..., \tag{5}$$

where $g(e_t)$ is a $J \times 1$ vector that satisfies the following assumption.

Assumption 2. $g(\cdot)$ is differentiable and satisfies the first-order Lipshitz condition

$\left|g_j'(x) - g_j'(y)\right| < M\left|x - y\right|$ for some constant M for all j, where $g_j(\cdot)$ is the j-th element of $g(\cdot)$. Also, $E\left[g(e_t)\right] = 0$, the second moment of $g(e_t)$ exists, and $E\left[g'(e_t)\right] < \infty$.

Define $C = E\left[g(e_t)g(e_t)'\right]$ and $D = E\left[g'(e_t)\right]$, and $\psi(e_t) = D'C^{-1}g(e_t)$, for $t = 1, 2, .., T$. Also define the correlation between e_t and $\psi(e_t)$ as:

$$\rho = \frac{\sigma_{\psi e}}{\sigma_\psi \sigma_e} \tag{6}$$

where $\sigma_\psi^2 = Var\left[\psi(e_t)\right] = Var\left[D'C^{-1}g(e_t)\right] = D'C^{-1}D$, and $\sigma_{\psi e} = E\left[\psi(e_t)e_t\right] = DC^{-1}E\left[g(e_t)e_t\right].$

We let $\tilde{\beta}_G$ denote the GMM estimator using the moments conditions (5) in the ADF regression (2). The asymptotic distributions of $\tilde{\beta}_G$ and its corresponding t-statistic are given as below.

Theorem 1. Suppose that a time series follows (1), and Assumptions 1 and 2 are satisfied.

Under the null hypothesis,

$$T\tilde{\beta}_G \Rightarrow \frac{a(1)}{\sigma_e \sigma_\psi} \left(\int_0^1 W_1(r)^2 dr \right)^{-1} \int_0^1 W_1(r) dW_2, \tag{7}$$

where $[W_1(r), W_2(r)]'$ is a bivariate Brownian motion with correlation ρ. We define the corresponding t-statistic as $t_G = \tilde{\beta}_G / se(\tilde{\beta}_G)$, where

$se(\tilde{\beta}_G) = \tilde{\sigma}_\psi^{-1} \sqrt{\left(\sum_{t=1}^T y_{t-1}^2 - \sum_{t=1}^T y_{t-1}\xi_t \left(\sum_{t=1}^T \xi_t \xi_t' \right)^{-1} \sum_{t=1}^T \xi_t' y_{t-1} \right)^{-1}}$, $\tilde{\sigma}_\psi^2 = \tilde{D}'\hat{C}^{-1}\tilde{D}$, $\tilde{D} = T^{-1} \sum_{t=1}^T g'(\tilde{e}_t)$, $\tilde{C} = T^{-1} \sum_{t=1}^T g(\tilde{e}_t) g(\tilde{e}_t)'$, and $\tilde{e}_t$ is the residual from GMM estimation of regression (2). Then, we have:

$$t_G \Rightarrow \rho DF + \sqrt{1-\rho^2} Z, \tag{8}$$

where ρ is defined in (6), DF denotes the Dickey-Fuller distribution as defined in (4), and Z signifies the standard normal distribution which is independent of DF.

proof. See the Appendix.

In the case where an intercept is allowed in the model, we use the regression:

$$\Delta y_t = c + \beta y_{t-1} + \sum_{j=1}^p \delta_j \Delta y_{t-j} + e_t, \; t = 1, 2, .., T, \tag{9}$$

and we have the additional moment conditions $E\left[g(e_t) \otimes (1, z_t)'\right] = 0$. In view of the expression for the estimator of β in (A.9) of the Appendix, this produces the GMM estimator that is given by:

$$T\tilde{\beta}_{G,\mu} = \left(\sigma_\psi^2 T^{-2} \sum_{t=1}^T \tilde{y}_{t-1}^2 \right)^{-1} T^{-1} \sum_{t=1}^T \tilde{y}_{t-1} \psi(e_t) + o_p(1),$$

where $\tilde{y}_{t-1} = y_{t-1} - T^{-1} \sum_{t=1}^T y_{t-1}$, $t = 1, 2, .., T$. Consequently, we have:

$$T\tilde{\beta}_{G,\mu} \Rightarrow \frac{a(1)}{\sigma_\psi \sigma_e} \int_0^1 \tilde{W}_1(r) dW_2(r) / \int_0^1 \tilde{W}_1(r)^2 dr, \tag{10}$$

where $\tilde{W}_1(r)$ is the demeaned Brownian motion: $\tilde{W}_1(r) = W_1(r) - \int_0^1 W_1(r)dr$. Also, by construction, we have

$$t_{G,\mu} \Rightarrow \rho DF_\mu + \sqrt{1-\rho^2}Z, \tag{11}$$

where DF_μ denotes the limiting distribution of the t-statistic from least squares in regression (9).

Similarly, when the model includes a linear time trend and an intercept, we use the regression

$$\Delta y_t = \alpha_1 + \alpha_2 t + \beta y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + e_t,\ t = 1, 2, .., T, \tag{12}$$

and this will result in the GMM estimator that follows

$$T\tilde{\beta}_{G,\tau} \Rightarrow \frac{a(1)}{\sigma_\psi \sigma_e} \left(\int_0^1 \breve{W}_1(r)^2 dr \right)^{-1} \int_0^1 \breve{W}_1(r) d\breve{W}_2(r), \tag{13}$$

where $\breve{W}(r)$ is the detrended Brownian motion. Also,

$$t_{G,\tau} \Rightarrow \rho DF_\tau + \sqrt{1-\rho^2}Z, \tag{14}$$

where DF_τ denotes the limiting distribution of the t-statistic for the OLS estimator of β in the regression (12).

Remark 1. Each of the asymptotic distributions of t_G, $t_{G,\mu}$, and $t_{G,\tau}$ depends on the nuisance parameter ρ. Hansen (1995) reports the critical values of the asymptotic distribution of these t-statistics for $\rho^2 = 0.1$ to 1.0, at increments of 0.1.

3 RALS Unit Root Tests

Now, we explain the RALS estimator. We first consider the model with an intercept as in (9), and use $x_t = (1, z_t')'$. We let $g(e_t) = \left(e_t, [h(e_t) - K]'\right)'$ and consider the moment condition $E[g(e_t) \otimes x_t] = 0$. We can split this moment condition into two parts. The first part is the usual moment conditions of least squares estimation:

$$E(e_t \otimes x_t) = 0. \tag{15}$$

The second part involves an additional $2(J-1)$ moment conditions given by:

$$E\left[\left(h\left(e_t\right)-K\right)\otimes x_t\right]=0. \tag{16}$$

Therefore, we have:

$$C=\begin{bmatrix} \sigma_e^2 & C_{21}' \\ C_{21} & C_{22} \end{bmatrix}, \text{ and } D=\begin{bmatrix} 1 \\ D_2 \end{bmatrix}, \tag{17}$$

where $C_{21}=E\left[e_t h(e_t)\right]$, $C_{22}=E\left[h(e_t)h(e_t)'\right]$, and $D_2=E\left[h'\left(e_t\right)\right]$.Then, we define:

$$\hat{w}_t=h(\hat{e}_t)-\hat{K}-\hat{e}_t\hat{D}_2,\ t=1,2,..,T, \tag{18}$$

where $\hat{e}_t$ is the OLS residual from the regression (9), $\hat{K}=\frac{1}{T}\sum_{t=1}^{T}h(\hat{e}_t)$, and $\hat{D}_2=\frac{1}{T}\sum_{t=1}^{T}h'(\hat{e}_t)$. The RALS-based testing equation is given by:

$$\Delta y_t=\alpha+\beta y_{t-1}+\sum_{j=1}^{p}\delta_j\Delta y_{t-j}+\hat{w}_t'\gamma+v_t,\ t=1,2,..,T. \tag{19}$$

The RALS estimator is obtained through least squares estimation applied to (19). We denote the estimator of β as $\tilde{\beta}_{R,\mu}$, and the corresponding t-statistic for $\beta=0$ is denoted as $t_{R,\mu}$. In the following, we show that the RALS estimator is asymptotically identical to the GMM estimator using moment conditions (15) and (16).

Theorem 2. Suppose that a time series follows (1) with $\phi=1$. Under Assumptions 1 and 2, the RALS estimator $\tilde{\beta}_{R,\mu}$ from (19) is asymptotically equivalent to the GMM estimator $\tilde{\beta}_{G,\mu}$ using the moment conditions (15) and (16). In addition, the limiting distributions of the RALS estimator $\tilde{\beta}_{R,\mu}$ and the t-statistic are the same as those of the corresponding GMM estimators.

proof. See the Appendix.

When a linear time trend is included in the regression, we use:

$$\Delta y_t=\alpha_1+\alpha_2 t+\beta y_{t-1}+\sum_{j=1}^{p}\delta_j\Delta y_{t-j}+\hat{w}_t'\gamma+v_t,\ t=1,2,..,T. \tag{20}$$

By construction, the RALS estimator of β and the corresponding t-statistic will have the same distributions as the corresponding GMM estimator and t-statistic as given in (13) and (14), respectively. In addition, we can obtain similar results for the RALS tests using a basic model without an intercept and a trend.

Next, we provide some guidance on how to apply the RALS procedure in practice.

- ρ^2 is estimated by

$$\hat{\rho}^2 = \hat{\sigma}_A^2 / \hat{\sigma}^2,$$

where $\hat{\sigma}^2$ is the usual estimate of the error variance in the standard ADF regression, and $\hat{\sigma}_A^2$ is the estimate of the error variance in the RALS regression in (19) and (20). See the proof of Theorem 2 [equations (A.16) and (A.19)]. Based on the estimated value $\hat{\rho}^2$, we can use the same critical values reported in Hansen (1995).

- When the sample size is small (e.g. $T \leq 50$), one may impose the restriction of $\beta = 0$ in the first step regression that yields the residuals for the augmented variables in $\hat{w}_t$. According to our simulations, this procedure significantly improves the size property of the test with only minimal effects on power. When the sample is relatively big (e.g., $T = 100$), this effect, however, disappears quickly.

4 Simulation Results

In this section, we investigate small sample properties of the RALS unit root tests. For example, suppose we have a testing regression $\Delta y_t = \alpha + \beta y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + e_t$, $t = 1, 2, .., T$. In the first step, we can take the residuals from this usual Dickey-Fuller regression and use them to construct $\hat{w}_t$ as a function of the residuals, $\hat{e}_t = \Delta y_t - \hat{\alpha} - \tilde{\beta} y_{t-1} - \sum_{j=1}^{p} \hat{\delta}_j \Delta y_{t-j}$. When the sample size is small, we estimate α and δ by imposing $\beta = 0$ and construct the augmented variable $\hat{w}_t$ from the residuals of the restricted regression. Then, in the second step, the t-statistic is computed following the augmented RALS regression: $\Delta y_t = \alpha + \beta y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + \gamma \hat{w}_t + v_t$. In our Monte Carlo study, we consider two RALS estimators, RALS(2&3) and RALS(t5), each of which is described in more detail below.

First, the estimator "RALS(2&3)" imposes the moment conditions that the second and third moments of the errors are not correlated with the lagged dependent variables. Therefore, we let $h(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3]'$. Letting $m_j = T^{-1} \sum_{t=1}^{T} \hat{e}_t^j$, for $j = 2, 3$, we have for RALS(2&3):

$$\hat{w}_t = \left[\hat{e}_t^2 - m_2,\ \hat{e}_t^3 - m_3 - 3m_2\hat{\varepsilon}_t\right]',\ t = 1, 2, .., T. \tag{21}$$

The moment condition $E\left[\left(e_t^2 - \sigma_e^2\right) y_{t-1}\right] = 0$ is the condition of no heteroskedasticity. This condition improves the efficiency of the estimator of β when the errors are not symmetric. The restriction on the third moments conditional on y_{t-1} improves efficiency unless $\mu_4 = 3\sigma^4$, where $\mu_j = E(e_t^j)$. In general, knowledge of higher moments μ_{j+1} are uninformative if $\mu_{j+1} = j\sigma^2\mu_{j-1}$. This is the redundancy condition identified by MaCurdy (1982) and Breusch *et al.* (1999). The normal distribution is the only distribution that satisfies the redundancy condition. Thus, if the distribution of the error term is not normal, the condition is not satisfied. In such cases, one may increase efficiency by augmenting $\hat{w}_t$ in the testing regression.

Second, the estimator "RALS(t5)" imposes the restrictions that arise from the score of the maximum likelihood procedure when the error density is assumed to be a t-distribution with 5 degrees of freedom. It may not be easy, perhaps, to justify using this particular density function for empirical applications. However, this density function is a popular choice for mimicking a fat-tailed distribution in the tests using the M-estimate for which a specific density function is assumed. Thus, RALS(t5) would achieve the efficiency gains when the distribution of the errors has fat-tails. In this case, we have $h(e_t) = (c+1)\, e_t/(c + e_t^2)$, and $D_2 = (c+1)\left(c - e_t^2\right)/(c + e_t^2)^2$ with $c = 5$. Therefore, in this scenario we have:

$$\hat{w}_t = \frac{6\hat{e}_t}{5 + \hat{e}_t^2} - \frac{1}{T}\sum_{t=1}^{T} \frac{6\hat{e}_t}{5 + \hat{e}_t^2} - \hat{e}_t \frac{1}{T}\sum_{t=1}^{T} \frac{6\left(5 - \hat{e}_t^2\right)}{(5 + \hat{e}_t^2)^2} \tag{22}$$

There is no compelling reason behind choosing $c = 5$. However, it seems that the tests are quite robust to the selection of different values of c. For example, our simulations that use $c = 3$, which are not reported here to save space, indicate that the empirical size and power of the tests are almost identical to the case when $c = 5$.To examine the size property, we report the rejection ratio for $\alpha = 0.05$ when $\phi = 1$. To examine the power, we use

$\phi = 0.9$. We simulated the sample cases for $T = 50$ and 100. All the results are based on 5,000 replications.

We compare RALS(2&3) and RALS(t5) with three other test statistics: (a) DF, the standard Dickey-Fuller test based on OLS; (b) AD, the test studied by Beelder (1996) and Shin and So (1999) based on adaptive estimation; and (c) M5, the test based on the M-estimate assuming that the true density is the student-t density with 5 degrees of freedom, as studied by Lucas (1995). We replicated the four distributions simulated by Shin and So (1999): (i) standard normal, (ii) t-distribution with $df = 3$, (iii) mixture normal: 0.5N(-3,1)+0.5N(3,1), and (iv) chi-square with $df = 1$.The simulation results for AD and M5 are reproduced from Shin and So (1999) for a comparison purpose.

Table 1 reports the results for the basic case when the errors, ε_t, in (1) are serially independent. The number of ADF augmentation terms (p) is set to zero. As is seen in Table 1, the sizes of the tests based on RALS(2&3) and RALS(t5) are quite close to the nominal 5% throughout. The size properties are generally better than AD or M5. When the error has a normal distribution, both RALS tests have correct sizes and the power is close to that of the DF tests. The power gain over the standard DF test is substantial when the errors are not normal. The overall power of RALS(2&3) and RALS(t5) compares favorably to the power of AD or M5 in most cases. The performance of RALS(t5) and M5 are similar when the true density is the student-t with 3 degrees of freedom, but RALS(t5) is more powerful when the density is mixture normal. When the true density is a chi-square distribution with one degree of freedom, RALS(2&3) dominates all other tests in terms of power. RALS(2&3) explicitly uses the moment condition that is useful when the error is not symmetric. This result may look surprising since the RALS moment conditions do not include the scores of the unknown log-density. But, the AD-based test does not seem to capture the possible efficiency gain from the non-symmetric feature of the error density. In our simulated distributions, RALS(t5) is marginally better than RALS(2&3) when the density is symmetric. However, as we can see for the case when the density is chi-square with one degree of freedom, RALS(2&3) is generally better than RALS (t5) when the error density is skewed. The difference in power is quite substantial in some cases.

In Tables 2 and 3, we compare the performance of the tests when the errors are serially correlated. In doing so, we compare only three tests, ADF, RALS(2&3) and RALS(t5), in two data generation processes:

$$AR : \varepsilon_t = 0.5\varepsilon_{t-1} + e_t,\ t = 1, 2, ...,$$

and

$$MA : \varepsilon_t = e_t - 0.5e_{t-1},\ t = 1, 2, ...$$

We report the size and power of the cases using a fixed ADF augmentation lag at $p = 2$ and $p = 4$ when $T = 50$, and $p = 3$ and $p = 6$ when $T = 100$. We also examine the cases when p is selected by various information criterion. We simulated the Akaike and Schwarz criteria, but report only the results from the Schwarz criterion since the results from the Akaike criterion were similar. The minimum and maximum values of p are set at 2 and 4 when $T = 50$, and at 3 and 6 when $T = 100$. We consider the case when the errors are generated from the standard normal, Cauchy, student-t distribution with 2 degrees of freedom, double exponential, chi-square distribution with 4 degrees of freedom, and beta(2,2) distribution. The Cauchy and the t-distribution with 2 degrees of freedom do not satisfy Assumptions 1 and 2, so we do not know the asymptotic distributions of the statistics in this case. However, it is interesting to see the performance of the tests in this situation. We report the results for the model with a constant term. To save space, we omit the results when a linear time trend is allowed, but the results are similar.

Table 2 presents the size and power properties of the ADF test, the RALS(2&3) test, and the RALS(t5) test when the errors follow an AR(1) model. The overall pattern of the results is similar. The sizes of all of the three tests are close to the 5% nominal size, even when the errors are generated from a Cauchy or t-distribution with two degrees of freedom. We note that the power difference between the two tests based on OLS and RALS is the greatest when the errors are generated from a Cauchy distribution. Also, as we observed in Table 1, RALS(t5) is more powerful than RALS(2&3) for all the symmetric distributions. However, RALS(2&3) is, in general, more powerful when the errors are asymmetric. In particular, the power of the RALS(t5)-based test is lower than that of the OLS-based tests when the error is chi-square distributed with 4 degrees of freedom; the power of RALS(2&3) is 63% while the power of RALS(t5) is 22% in the model without time trend for $T = 100$ and $p = 3$.

Table 3 presents the size and power properties of the three tests when the errors follow an MA(1) model. When $T = 50$ and p is determined by the Schwarz criteria or when p is fixed at 2, all of the tests tend to over-reject the null hypothesis. However, when $p = 4$ (for $T = 50$), the size of all three tests is quite close to the 5% nominal size. By comparison, when $T = 100$ the size of all three tests is reasonably close to the 5% nominal size. Thus the overall size of both the RALS(2&3) and RALS(t5) tests seem as robust as the size of the traditional ADF test. With regard to the power of the tests, we observe a pattern similar to what was found in the case of AR(1) errors. Except for the case where the errors follow a normal distribution, the RALS-based tests are substantially more powerful than the OLS-based ADF tests, and the RALS(2&3) test compares favorably with the RALS(t5) test.

We next examine the degree to which our RALS-based unit root tests are robust to various forms of non-linearity. The RALS-based unit root tests exhibit efficiency gains with non-normal errors but these tests are not designed specifically to detect non-linearity. However, we presume that some forms of non-linearity could be captured in non-normal errors. Thus, we are interested in examining how much power the RALS-based tests have against various forms of non-linearity. For this purpose, we follow the work of Choi and Moh (2007) who considered sixteen non-linear model specifications and examined the power of four popular non-linear unit root tests: the KSS test of Kapetanios et al. (2003) using an ESTAR model; the sign test of So and Shin (2001); the M-TAR tests of Enders and Granger (1998); and the Inf-t test of Park and Shintani (2005). To illustrate the data generating process of these nonlinear models, we present the plot of the Gaussian kernel density function in Figure 1, for each of these sixteen non-linear models. Using these various nonlinear model specifications, we wish to examine the power of our RALS-based tests and compare with four popular nonlinear tests as well as the traditional DF test. In this case, we let $T = 50$ and 100 and we use the 10% significance level (results for the KSS, Sign, MTAR and Inf-t tests are reproduced from Table 2 of Choi and Moh, 2008, p. 90, $\rho_1 = \rho_2 = 0.9$). The results in Table 4 provide encouraging implications for our RALS-based tests: they are more powerful than the other unit root tests in 14 out of 16 cases. The difference in power between the RALS-based tests and the other tests is significant in most cases. The difference in power is negligible in the two remaining cases where the RALS-based tests are less powerful than at least one of other tests (DGP 4 and 7). In short, although most of the non-linear tests generally are not powerful against other forms of non-linearity, the RALS-based tests are fairly robust to various forms of non-linearity, except for a few cases. All tests including the RALS-based tests suffer from a loss of power when structural changes occur in the data (DGP 14 and 15). However, this result simply confirms the initial finding of Perron (1989) who noted that unit root tests will lose power if existing breaks are ignored. Thus, in such cases where the models are mis-specified, all tests will be subject to size distortions and/or loss of power.

Overall, our simulation results show that the RALS-based unit root tests remain powerful under various forms of non-normal errors and under many non-linear alternatives.

5 An Application of the RALS Unit Root Test

We now present an empirical application of our new test by applying the RALS-based unit root test "RALS(2&3)" to the CPI inflation rate series of several OECD countries. Knowledge of the long-run properties of the inflation rate (or the aggregate price level) is a key component for policy makers, applied econometricians and financial analysts who seek to understand or affect the behavior of the macroeconomy. For example, forecasters who seek to project expected or future inflation rates must know whether or not inflation rates are stationary when building their models. Similarly, officials who seek to use monetary policy to affect the behavior of macroeconomic variables also must have knowledge of the long-run properties of inflation when constructing optimal commodity price rules or when engaging in inflation rate targeting. In addition, financial planners who, for example, rely on the capital asset pricing model also must understand the long-run behavior of inflation.

Yet the question of whether or not the inflation rate is stationary still is widely disputed in the literature. Numerous researchers, employing various methodologies applied to the inflation rates of several different countries, have found this series to be non-stationary (see, for example, Crowder and Hoffman (1996), Rapach and Weber (2004), Crowder and Phengpis (2007)). At the same time, several authors have concluded that inflation is stationary (see, for example, Baillie, Chung and Tieslau (1996), and Costantini and Lupi (2007)). This contradiction in the empirical results on the inflation rate might be due, in part, to the low power of traditional unit root tests. We wish to examine whether or not accounting for non-normality in the series will make a difference. Since our test will be more powerful in the face of departures from normality or apparent non-linearities, we seek to shed light on the issue of whether or not inflation is stationary through the application of our more powerful tests.

The series used in our analysis are the first-differences of the log of the monthly consumer price index series (all items) for 12 OECD countries.[1] The data were taken from the International Monetary Fund's "International Financial Statistics" CD rom (July 2009), and span the period from January of 1957 through April of 2009. We analyze these inflation rates applying the RALS(2&3) test from both (19) and (20). The first step of the procedure begins by conducting the traditional Dickey-Fuller unit root test while choosing the optimal number of augmentation terms to ensure non-correlated errors in the testing equation.[2] The OLS residuals from this equation are then retained for use in the second step. The second step involves estimation of the RALS unit root testing equation, which is an augmented version of the original Dickey-Fuller equation.

The results of the RALS unit root test are presented in table 5. In the case where the testing equation includes only an intercept, the RALS unit root test rejects the null of a

[1] The countries are: Belgium, Canada, Finland, France, Italy, Japan, Luxembourg, the Netherlands, Norway, Spain, the UK and the USA.

[2] One may choose the optimal lag length following the usual practice. For example, one can determine the optimal number of augmentation terms using the sequential t-test, following Ng and Perron (1995), or through use of the traditional Akaike Information Criteria or Schwartz Criteria, or other similar methods. In our application, we followed the procedure of Ng and Perron (1995) with a maximum of 12 lags.

unit root in 8 out of 12 cases, while the Dickey-Fuller unit root test rejects the null in only 3 of 12 cases. Similarly, when allowing for both a constant and a trend in the testing equation, the RALS unit root test rejects the null in 7 of 12 cases, while the Dickey-Fuller test rejects the null in only 2 cases. The ability of the RALS unit root test to reject the null in significantly more cases lends support to the notion that our test is better able to distinguish non-normality from non-stationarity.

6 Concluding Remarks

This paper proposes new unit root tests that are more powerful when the error term follows a non-normal distribution. The improved power is gained by utilizing more moment conditions through a computationally simple procedure. Specifically, we extend the residual augmented least squares (RALS) estimator proposed by Im and Schmidt (2008) in order to use the information implied by non-normal errors in testing for a unit root hypothesis. We show that the asymptotic distribution of our simple RALS-based estimator is the same as that of the GMM estimator. Our Monte Carlo simulation results show that the size of the RALS-based unit root tests is quite close to the asymptotic size, and the power is improved significantly over the usual Dickey-Fuller tests when the error is not normal. As such, our findings show significant efficiency gains when the information on non-normality is utilized, although this information is ignored in traditional unit root tests. In addition, it is encouraging to see that the RALS-based unit root tests remain powerful under many cases of non-linear alternatives, when compared with other popular non-linear unit root tests. While it still is desirable to obtain the best possible model specifications, this paper shows that we can achieve increased power by utilizing the additional information on non-normal errors, if any, which a time series itself contains.

A Appendix

Lemma A1. We let $z_t = (y_{t-1}, \xi_t')'$, as defined previously in equation (5). We define a $(p+1) \times (p+1)$ matrix, $\Upsilon_T = diag\left(T, \sqrt{T}, ..., \sqrt{T}\right)$. Assume that Assumptions 1 and 2. Then, we have under the null hypothesis:

$$\sum_{t=1}^{T} \left[g'(e_t) \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right] \Rightarrow D \otimes \int zz', \tag{A.1}$$

$$\sum_{t=1}^{T} g(e_t) g(e_t)' \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1} \Rightarrow C \otimes \int zz', \tag{A.2}$$

where $\int zz' = diag\left(a(1)^{-2}\sigma_e^2 \int_0^1 W_1(r)^2 dr, E(\xi_t \xi_t')\right)$, and C and D are defined in (17). Also, we have:

$$\sum_{t=1}^{T} \psi(e_t) \Upsilon_T^{-1} z_t = \begin{bmatrix} T^{-1} \sum_{t=1}^{T} \psi(\varepsilon_t) y_{t-1} \\ T^{-1/2} \sum_{t=1}^{T} \psi(\varepsilon_t) \xi_t \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sigma_\psi \sigma_\varepsilon}{a(1)} \int_0^1 W_1(r) dW_2(r) \\ \Gamma \end{bmatrix}, \tag{A.3}$$

where Γ is a $p \times p$ multivariate normal variable with covariance matrix $\sigma_\psi^2 E(\xi_t \xi_t')$.

proof. Lucas (1995, Lemma 1 in Appendix). See also Hansen (1995, Lemma).

Lemma A2. ρ is defined as in equation (6). Then,

$$\rho = \frac{1}{\sigma_e \sigma_\psi}. \tag{A.4}$$

Also,

$$\frac{1}{\sigma_\psi^2} = \sigma_e^2 - \left(C_{21} - \sigma_e^2 D_2\right)' \left(C_{22} + \sigma_e^2 D_2 D_2' - C_{21} D_2' - D_2 C_{21}'\right)^{-1} \left(C_{21} - \sigma_e^2 D_2\right). \tag{A.5}$$

proof. The first result follows from a routine matrix algebra using the partitioned inverse lemma. For the second result, a straightforward algebra gives:

$$\left(D' C^{-1} D\right)^{-1} = \sigma_e^2 \left(1 + \left(C_{21} - \sigma_e^2 D_2\right)' \left(\sigma_e^2 C_{22} - C_{21} C_{21}'\right)^{-1} \left(C_{21} - \sigma_e^2 D_2\right)\right)^{-1},$$

which is the same as $1/\sigma_\psi^2$; see Amemiya (1985, p. 461, Lemma 20).

PROOF OF THEOREM 1: We note that the entire proof follows immediately from Lucas (1995, Theorem 1) since the GMM estimator is obtained by solving the score $\sum_{t=1}^{T}\left[DC^{-1}g(e_t)z_t\right] = \sum_{t=1}^{T}\left[\psi(e_t)z_t\right] = 0$, and this score could be viewed as that of the M-estimate. Here, we provide more details. Let $\theta = (\beta, \delta_1, \delta_2, ..., \delta_p)'$. The GMM estimator is obtained by solving:

$$\min_{\theta} \sum_{t=1}^{T} \left[g(e_t) \otimes z_t\right]' \hat{\Lambda}^{-1} \sum_{t=1}^{T} \left[g(e_t) \otimes z_t\right], \tag{A.6}$$

where $\hat{\Lambda} = \left(\sum_{t=1}^{T} g(\hat{e}_t)g(\hat{e}_t)' \otimes z_t z_t'\right)$, and $\hat{e}_t$ is the residual from an initial consistent estimator of θ. Taking the derivative with respect to θ, we obtain the score:

$$\sum_{t=1}^{T} \left[g'(\tilde{e}_t) \otimes z_t z_t'\right]' \hat{\Lambda}^{-1} \sum_{t=1}^{T} \left[g(\tilde{e}_t) \otimes z_t\right] = 0, \tag{A.7}$$

where $\tilde{e}_t = \Delta y_t - z_t\tilde{\theta}$, and $\tilde{\theta}$ is the GMM estimator. The Taylor series expansion of the term $\sum_{t=1}^{T}\left[g(\tilde{e}_t) \otimes z_t\right]$ with respect to the true disturbance e_t and premultiplication of $I_J \otimes \Upsilon_T^{-1}$ yields:

$$\begin{aligned} &\sum_{t=1}^{T} \left[g(\tilde{e}_t) \otimes \Upsilon_T^{-1} z_t\right] \\ =\; &\sum_{t=1}^{T} \left[g(e_t) \otimes \Upsilon_T^{-1} z_t - g'(e_t) \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1} \Upsilon_T \left(\tilde{\theta} - \theta\right)\right] + o_p(1). \end{aligned} \tag{A.8}$$

Solving (A.7) with respect to $\Upsilon_T\left(\tilde{\theta} - \theta\right)$, after substituting (A.8) into (A.7), we obtain:

$$\begin{aligned} &\Upsilon_T(\tilde{\theta} - \theta) = \\ &\left\{ \sum_{t=1}^{T} \left[g'(\tilde{e}_t) \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right]' \left[\sum_{t=1}^{T} g(\hat{e}_t)g(\hat{e}_t)' \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right]^{-1} \sum_{t=1}^{T} \left[g'(e_t) \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right] \right\}^{-1} \\ &\times \left\{ \sum_{t=1}^{T} \left[g'(\tilde{e}_t) \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right]' \left[\sum_{t=1}^{T} g(\hat{e}_t)g(\hat{e}_t)' \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right]^{-1} \sum_{t=1}^{T} \left[g(e_t) \otimes \Upsilon_T^{-1} z_t\right] \right\} + o_p(1). \end{aligned} \tag{A.9}$$

Noting that:

$$\sum_{t=1}^{T} \left\{\left[g'(\tilde{e}_t) - g'(e_t)\right] \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right\} = o_p(1),$$

and:

$$\sum_{t=1}^{T} \left\{\left[g(\hat{e}_t)g(\hat{e}_t)' - g(e_t)g(e_t)'\right] \otimes \Upsilon_T^{-1} z_t z_t' \Upsilon_T^{-1}\right\} = o_p(1),$$

we have, from Lemma A1:

$$T\tilde{\beta}_G \Rightarrow \frac{a(1)}{\sigma_\psi \sigma_e}\left(\int_0^1 W_1(r)^2 dr\right)^{-1}\int_0^1 W_1(r)dW_2(r), \tag{A.10}$$

where $[W_1(r), W_2(r)]$ is a bivariate Brownian motion with correlation ρ. Then, we have for the t-statistic:

$$t_G \Rightarrow \left(\int_0^1 W_1(r)^2 dr\right)^{-1/2}\int_0^1 W_1(r)dW_2(r), \tag{A.11}$$

which is a mixture of the Dickey-Fuller and the standard normal distribution described in (8). To see this, note:

$$T^{-1/2}\sum_{t=1}^{[rT]}\begin{bmatrix} e_t \\ \psi(e_t) \end{bmatrix} \Rightarrow \begin{bmatrix} \sigma_e W_1(r) \\ \sigma_\psi W_2(r) \end{bmatrix}, \tag{A.12}$$

where $[rT]$ denotes the integer part of rT. Therefore:

$$W_2(r) = \rho W_1(r) + \sqrt{1-\rho^2}W_3(r), \tag{A.13}$$

where $W_3(r)$ is independent of $W_1(r)$. The result follows if we note that:

$$\left(\int_0^1 W_1(r)^2 d(r)\right)^{-1/2}\int_0^1 W_1(r)dW_3(r)$$

is standard normal.

PROOF OF THEOREM 2: Define a variable as a function of true disturbances:

$$w_t = h(e_t) - K - e_t D_2,\ t = 1, 2, .., T.$$

The variables in w_t are not observable, but we momentarily assume that they are observed. Then we show that the augmentation of w_t or $\hat{w}_t$ asymptotically yields the same estimator of $T\beta$. Consider a regression:

$$\Delta y_t = \alpha_1 + \beta y_{t-1} + \sum_{j=1}^{p}\delta_j \Delta y_{t-j} + w_t'\gamma + v_t,\ t = 1, 2, .., T. \tag{A.14}$$

Therefore,

$$e_t = w_t'\gamma + v_t,\ t = 1, 2, ... \tag{A.15}$$

Let $\hat{\beta}_A^*$ be the least squares estimator of β from regression (A.14), $\sigma_v^2 = Var(v_t)$, and:

$$\lambda = \frac{\sigma_{ev}}{\sigma_e \sigma_v} = \frac{\sigma_v}{\sigma_e}, \tag{A.16}$$

where $\sigma_{ev} = E(\varepsilon_t v_t)$. The second equality of (A.16) follows since w_t and v_t are not correlated, so that $\sigma_{ev} = \sigma_v^2$. From Hansen (1995, Theorem 2 and 3), we have:

$$T\hat{\beta}_A^* \Rightarrow \frac{\sigma_v}{\sigma_e}\left(\int_0^1 W_4(r)^2\right)^{-1}\int_0^1 W_4(r)dW_5(r), \tag{A.17}$$

and for the t-statistic:

$$t_A^* = \lambda DF_\mu + \sqrt{1-\lambda^2}N(0,1), \tag{A.18}$$

where $[W_4(r), W_5(r)]'$ is the bivariate Brownian motion with correlation λ. Next, we will show that:

$$\rho = \lambda. \tag{A.19}$$

Note that $\gamma = E(w_t w_t')^{-1}E(w_t e_t)$, so we have:

$$\sigma_v^2 = \sigma_e^2 - E(e_t w_t')E(w_t w_t')^{-1}E(w_t e_t). \tag{A.20}$$

Also, $E(w_t e_t) = C_{21} - \sigma_e^2 D_2$ and $E(w_t w_t') = C_{22} + \sigma_\varepsilon^2 D_2 D_2' - C_{21}D_2' - D_2 C_{21}'$. Therefore,

$$\sigma_v^2 = \sigma_e^2 - \left(C_{21} - \sigma_\varepsilon^2 D_2\right)'\left(C_{22} + \sigma_\varepsilon^2 D_2 D_2' - C_{21}D_2' - D_2 C_{21}'\right)^{-1}\left(C_{21} - \sigma_\varepsilon^2 D_2\right),$$

which becomes $1/\sigma_\psi^2$ from Lemma A1. Therefore, we have $\rho = \lambda$.

Now we let $\hat{\beta}_A$ be the OLS estimator of β in the regression (19). The proof is complete if we show that $T\hat{\beta}_A$ and $T\hat{\beta}_A^*$ are identical asymptotically. Let $\hat{\zeta}_t = \left(\tilde{\xi}_t', \hat{w}_t'\right)'$, where $\tilde{\xi}_t = \xi_t - T^{-1}\sum_{t=1}^T \xi_t$. Then we have:

$$T\hat{\beta}_A = \frac{T^{-1}\left(\sum_{t=1}^T \tilde{y}_{t-1}e_t - \sum_{t=1}^T \tilde{y}_{t-1}\zeta_t'\left(\sum_{t=1}^T \tilde{\zeta}_t\tilde{\zeta}_t'\right)^{-1}\sum_{t=1}^T \tilde{\zeta}_t e_t\right)}{T^{-2}\left(\sum_{t=1}^T \tilde{y}_{t-1}^2 - \sum_{t=1}^T \tilde{y}_{t-1}\zeta_t'\left(\sum_{t=1}^T \tilde{\zeta}_t\tilde{\zeta}_t'\right)^{-1}\sum_{t=1}^T \zeta_t\tilde{y}_{t-1}\right)},$$

Since $T^{-1}\sum_{t=1}^T \hat{w}_t\xi_t' = o_p(1)$, and $T^{-1}\sum_{t=1}^T \tilde{\xi}_t e_t = o_p(1)$, we have:

$$T\hat{\beta}_A = \frac{T^{-1}\left(\sum_{t=1}^T \tilde{y}_{t-1}e_t - \sum_{t=1}^T \tilde{y}_{t-1}\hat{w}_t'\left(\sum_{t=1}^T \hat{w}_t\hat{w}_t'\right)^{-1}\sum_{t=1}^T \hat{w}_t' e_t\right)}{T^{-2}\left(\sum_{t=1}^T \tilde{y}_{t-1}^2\right)} + o_p(1).$$

Similarly,

$$T\hat{\beta}_A^* = \frac{T^{-1}\left(\sum_{t=1}^T \tilde{y}_{t-1}e_t - \sum_{t=1}^T \tilde{y}_{t-1}w_t'\left(\sum_{t=1}^T \tilde{w}_t\tilde{w}_t'\right)^{-1}\sum_{t=1}^T \tilde{w}_t'e_t\right)}{T^{-2}\left(\sum_{t=1}^T \tilde{y}_{t-1}^2\right)} + o_p(1).$$

$T\hat{\beta}_A$ and $T\hat{\beta}_A^*$ are asymptotically identical if $T^{-1}\sum \tilde{y}_{t-1}\left(\hat{w}_t - w_t\right) = o_p\left(1\right)$. However:

$$T^{-1}\sum \tilde{y}_{t-1}\hat{w}_t = T^{-1}\sum \tilde{y}_{t-1}\left[h(\varepsilon_t) + (\hat{\varepsilon}_t - \varepsilon_t)h'\left(\varepsilon_t\right) - \hat{\varepsilon}_t\hat{D}_2\right] + o_p(1)$$

Therefore,

$$\begin{aligned} & T^{-1}\sum \tilde{y}_{t-1}\left(\hat{w}_t - w_t\right) \qquad \text{(A.21)} \\ = \; & T^{-1}\sum \tilde{y}_{t-1}\left[(\hat{\varepsilon}_t - \varepsilon_t)h'\left(\varepsilon_t\right) - (\hat{\varepsilon}_t - \varepsilon_t)\hat{D}_2 - \varepsilon_t\left(\hat{D}_2 - D_2\right)\right] + o_p(1) \end{aligned}$$

but,

$$T^{-1}\sum \tilde{y}_{t-1}(\hat{\varepsilon}_t - \varepsilon_t)h'\left(\varepsilon_t\right) = T\left(\hat{\beta} - \beta\right)T^{-2}\sum \tilde{y}_{t-1}^2 h'(\varepsilon_t) + o_p(1), \qquad \text{(A.22)}$$

$$T^{-1}\sum \tilde{y}_{t-1}(\hat{\varepsilon}_t - \varepsilon_t)\hat{D}_2 = \hat{D}_2 T\left(\hat{\beta} - \beta\right)T^{-2}\sum \tilde{y}_{t-1}^2 + o_p(1), \qquad \text{(A.23)}$$

and:

$$T^{-1}\sum \tilde{y}_{t-1}\varepsilon_t\left(\hat{D}_2 - D_2\right) = o_p(1). \qquad \text{(A.24)}$$

The two terms (A.22) and (A.23) cancel each other in the limit in (A.21), so the proof is complete.

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Table 1
Rejection Ratio of Various Tests
No Serial Correlations, 5% significance level

No Time Trend

Distributions		DF (*T*=50)	AD (*T*=50)	M5 (*T*=50)	RALS (2&3) (*T*=50)	RALS (t5) (*T*=50)	DF (*T*=100)	AD (*T*=100)	M5 (*T*=100)	RALS (2&3) (*T*=100)	RALS (t5) (*T*=100)
Normal	$\phi = 1$	0.060	0.043	0.094	0.054	0.059	0.051	0.049	0.069	0.051	0.051
	$\phi = 0.9$	0.146	0.091	0.198	0.132	0.136	0.352	0.263	0.346	0.311	0.330
Student t	$\phi = 1$	0.058	0.045	0.052	0.051	0.050	0.053	0.067	0.037	0.051	0.051
df=3	$\phi = 0.9$	0.139	0.197	0.291	0.270	0.296	0.358	0.535	0.649	0.615	0.676
Mixture	$\phi = 1$	0.055	0.040	0.178	0.044	0.045	0.058	0.049	0.130	0.045	0.044
Normal	$\phi = 0.9$	0.145	0.790	0.217	0.850	0.916	0.361	0.991	0.281	0.995	0.998
Chi-square	$\phi = 1$	0.046	0.048	0.058	0.043	0.045	0.052	0.047	0.036	0.041	0.045
df=1	$\phi = 0.9$	0.126	0.360	0.332	0.909	0.339	0.355	0.796	0.666	0.999	0.723

With Linear Trend

Distributions		DF (*T*=50)	AD (*T*=50)	M5 (*T*=50)	RALS (2&3) (*T*=50)	RALS (t5) (*T*=50)	DF (*T*=100)	AD (*T*=100)	M5 (*T*=100)	RALS (2&3) (*T*=100)	RALS (t5) (*T*=100)
Normal	$\phi = 1$	0.062	0.025	0.148	0.054	0.057	0.057	0.035	0.078	0.054	0.055
	$\phi = 0.9$	0.109	0.049	0.204	0.092	0.099	0.216	0.129	0.251	0.191	0.206
Student t	$\phi = 1$	0.064	0.026	0.062	0.060	0.052	0.054	0.039	0.036	0.049	0.049
df=3	$\phi = 0.9$	0.100	0.120	0.231	0.187	0.192	0.197	0.386	0.495	0.441	0.507
Mixture	$\phi = 1$	0.054	0.024	0.292	0.042	0.044	0.053	0.027	0.192	0.042	0.043
Normal	$\phi = 0.9$	0.097	0.628	0.258	0.669	0.784	0.219	0.981	0.255	0.981	0.996
Chi-square	$\phi = 1$	0.055	0.026	0.064	0.045	0.051	0.055	0.038	0.048	0.039	0.049
df=1	$\phi = 0.9$	0.081	0.251	0.277	0.797	0.224	0.202	0.647	0.506	0.991	0.529

The 5% significance level was used. AD denotes the test based on the adaptive MLE of Shin and So (1999) and M5 is the test of Lucas (1995) using the M-estimate assuming that the error density is the student-*t* with 5 degrees of freedom. Mixture normal is 0.5N(-3,1) + 0.5N(3,1). All the figures for AD and M5 have been reproduced from Shin and So (1999).

Table 2
Rejection Ratio of Various Tests
AR(1) error with AR coefficient 0.5 (No Time Trend)

T = 50

Distributions		ADF			RALS(2&3)			RALS(t5)		
		p=2	p=4	SC	p=2	p=4	SC	p=2	p=4	SC
Normal	$\phi = 1$	0.056	0.055	0.071	0.053	0.054	0.061	0.054	0.053	0.067
	$\phi = 0.9$	0.100	0.080	0.116	0.087	0.074	0.097	0.093	0.073	0.110
Cauchy	$\phi = 1$	0.075	0.079	0.055	0.046	0.053	0.063	0.048	0.059	0.051
	$\phi = 0.9$	0.074	0.074	0.088	0.664	0.599	0.698	0.567	0.504	0.568
Student t	$\phi = 1$	0.063	0.060	0.056	0.050	0.054	0.064	0.049	0.056	0.055
df=2	$\phi = 0.9$	0.080	0.069	0.089	0.300	0.252	0.326	0.343	0.281	0.341
Double	$\phi = 1$	0.051	0.053	0.059	0.048	0.050	0.057	0.049	0.051	0.058
Exponential	$\phi = 0.9$	0.091	0.084	0.109	0.131	0.110	0.150	0.151	0.120	0.161
Chi-square	$\phi = 1$	0.051	0.058	0.061	0.050	0.045	0.032	0.052	0.051	0.061
4 df	$\phi = 0.9$	0.094	0.080	0.110	0.260	0.202	0.191	0.091	0.081	0.106
Beta(2,2)	$\phi = 1$	0.060	0.055	0.073	0.057	0.050	0.059	0.053	0.047	0.060
	$\phi = 0.9$	0.100	0.087	0.121	0.126	0.101	0.133	0.131	0.103	0.149

T = 100

Distributions		ADF			RALS(2&3)			RALS(t5)		
		p=3	p=6	SC	p=3	p=6	SC	p=3	p=6	SC
Normal	$\phi = 1$	0.055	0.053	0.061	0.056	0.048	0.052	0.055	0.051	0.056
	$\phi = 0.9$	0.217	0.163	0.243	0.196	0.142	0.217	0.207	0.145	0.230
Cauchy	$\phi = 1$	0.080	0.076	0.055	0.040	0.042	0.055	0.045	0.050	0.042
	$\phi = 0.9$	0.144	0.125	0.181	0.907	0.852	0.943	0.796	0.775	0.803
Student t	$\phi = 1$	0.053	0.053	0.050	0.049	0.052	0.067	0.050	0.047	0.049
df=2	$\phi = 0.9$	0.190	0.134	0.220	0.610	0.512	0.678	0.716	0.616	0.740
Double	$\phi = 1$	0.059	0.053	0.062	0.055	0.050	0.063	0.055	0.047	0.056
Exponential	$\phi = 0.9$	0.216	0.155	0.246	0.321	0.231	0.362	0.377	0.273	0.400
Chi-square	$\phi = 1$	0.052	0.048	0.052	0.046	0.048	0.025	0.050	0.045	0.047
df=4	$\phi = 0.9$	0.224	0.156	0.242	0.629	0.480	0.556	0.217	0.157	0.237
Beta(2,2)	$\phi = 1$	0.057	0.053	0.063	0.048	0.048	0.052	0.050	0.046	0.054
	$\phi = 0.9$	0.216	0.155	0.246	0.324	0.225	0.355	0.343	0.235	0.376

Table 3
Rejection Ratio of Various Tests
MA(1) error with MA coefficient –0.5 (No Time Trend)

<u>$T = 50$</u>

Distributions		ADF			RALS(2&3)			RALS(t5)		
		p=2	p=4	SC	p=2	p=4	SC	p=2	p=4	SC
Normal	$\phi = 1$	0.088	0.054	0.095	0.079	0.051	0.079	0.085	0.052	0.091
	$\phi = 0.9$	0.227	0.109	0.231	0.190	0.091	0.182	0.205	0.097	0.208
Cauchy	$\phi = 1$	0.102	0.077	0.079	0.180	0.092	0.194	0.109	0.068	0.108
	$\phi = 0.9$	0.163	0.087	0.177	0.853	0.712	0.867	0.625	0.557	0.622
Student t	$\phi = 1$	0.091	0.060	0.079	0.109	0.060	0.123	0.102	0.059	0.101
df=2	$\phi = 0.9$	0.198	0.089	0.199	0.538	0.347	0.547	0.553	0.379	0.539
Double	$\phi = 1$	0.083	0.053	0.085	0.088	0.050	0.094	0.087	0.054	0.090
Exponential	$\phi = 0.9$	0.219	0.104	0.216	0.285	0.149	0.289	0.320	0.168	0.313
Chi-square	$\phi = 1$	0.085	0.050	0.088	0.108	0.054	0.070	0.079	0.051	0.082
df=4	$\phi = 0.9$	0.223	0.102	0.227	0.522	0.282	0.425	0.209	0.098	0.212
Beta(2,2)	$\phi = 1$	0.095	0.054	0.101	0.086	0.051	0.084	0.087	0.050	0.090
	$\phi = 0.9$	0.237	0.112	0.241	0.259	0.135	0.250	0.276	0.140	0.277

<u>$T = 100$</u>

Distributions		ADF			RALS(2&3)			RALS(t5)		
		p=3	p=6	SC	p=3	p=6	SC	p=3	p=6	SC
Normal	$\phi = 1$	0.050	0.049	0.057	0.052	0.046	0.053	0.050	0.046	0.056
	$\phi = 0.9$	0.260	0.187	0.287	0.230	0.172	0.257	0.246	0.176	0.269
Cauchy	$\phi = 1$	0.078	0.073	0.049	0.041	0.044	0.057	0.040	0.042	0.036
	$\phi = 0.9$	0.171	0.134	0.207	0.938	0.889	0.962	0.785	0.776	0.796
Student t	$\phi = 1$	0.053	0.048	0.048	0.050	0.047	0.065	0.048	0.046	0.050
df=2	$\phi = 0.9$	0.235	0.159	0.266	0.682	0.570	0.745	0.771	0.666	0.798
Double	$\phi = 1$	0.058	0.051	0.059	0.052	0.050	0.058	0.053	0.048	0.054
Exponential	$\phi = 0.9$	0.263	0.187	0.303	0.379	0.280	0.444	0.440	0.321	0.486
Chi-square	$\phi = 1$	0.047	0.044	0.050	0.046	0.045	0.027	0.041	0.043	0.045
df=4	$\phi = 0.9$	0.254	0.183	0.293	0.705	0.543	0.650	0.248	0.174	0.286
Beta(2,2)	$\phi = 1$	0.051	0.047	0.059	0.047	0.046	0.053	0.049	0.045	0.052
	$\phi = 0.9$	0.253	0.186	0.296	0.372	0.265	0.407	0.391	0.280	0.438

Table 4
Rejection Ratio of Various Tests
Under Various Nonlinear Models (No Time Trend)

DGP	Model	DF	KSS	Sign	MTAR	Inf-t	RALS (2&3)	RALS (t5)
					T = 50			
1	AR(1)	0.29	0.19	0.19	0.08	0.09	0.411	0.376
2	Generalized AR(1)	0.29	0.19	0.19	0.08	0.19	0.411	0.376
3	Bilinear	0.29	0.22	0.17	0.10	0.25	0.425	0.388
4	Nonlinear AR	0.95	0.59	0.99	1.00	1.00	0.897	0.903
5	Squared Relation	0.69	0.44	0.59	0.50	0.66	0.822	0.695
6	Exponential Relation	0.76	0.76	0.67	0.67	0.76	0.853	0.795
7	Bilinear	0.95	0.59	0.99	1.00	1.00	0.897	0.901
8	SETAR(1)	0.22	0.15	0.16	0.06	0.14	0.372	0.328
9	EQ-TAR	0.22	0.16	0.17	0.06	0.16	0.359	0.316
10	Band TAR	0.19	0.12	0.12	0.04	0.13	0.319	0.262
11	ESTAR	0.19	0.13	0.14	0.05	0.13	0.332	0.293
12	LSTAR	0.50	0.26	0.35	0.25	0.37	0.577	0.551
13	Markov-switching	0.46	0.30	0.41	0.24	0.37	0.505	0.510
14	Level Shift	0.08	0.08	0.08	0.02	0.06	0.144	0.114
15	Multiple Shift	0.00	0.00	0.05	0.01	0.00	0.008	0.004
16	Variance Shift	0.30	0.28	0.23	0.09	0.30	0.470	0.395
					T = 100			
1	AR(1)	0.52	0.37	0.47	0.21	0.42	0.590	0.569
2	Generalized AR(1)	0.52	0.37	0.46	0.21	0.41	0.590	0.569
3	Bilinear	0.56	0.40	0.39	0.20	0.54	0.577	0.547
4	Nonlinear AR	0.99	0.77	1.00	1.00	1.00	0.968	0.973
5	Squared Relation	0.86	0.60	0.88	0.81	0.89	0.949	0.863
6	Exponential Relation	0.76	0.82	0.84	0.84	0.82	0.97	0.920
7	Bilinear	0.99	0.77	1.00	1.00	1.00	0.968	0.973
8	SETAR(1)	0.36	0.28	0.36	0.10	0.28	0.442	0.423
9	EQ-TAR	0.33	0.34	0.33	0.10	0.32	0.409	0.398
10	Band TAR	0.21	0.18	0.24	0.05	0.17	0.307	0.284
11	ESTAR	0.25	0.21	0.29	0.07	0.20	0.343	0.313
12	LSTAR	0.84	0.46	0.69	0.66	0.75	0.803	0.791
13	Markov-switching	0.77	0.49	0.75	0.57	0.70	0.727	0.775
14	Level Shift	0.08	0.12	0.19	0.03	0.09	0.121	0.097
15	Multiple Shift	0.00	0.00	0.24	0.00	0.00	0.000	0.000
16	Variance Shift	0.47	0.41	0.48	0.25	0.54	0.587	0.536

The 10% significance level was used in this table.

Table 5
Empirical Application of Inflation Rates

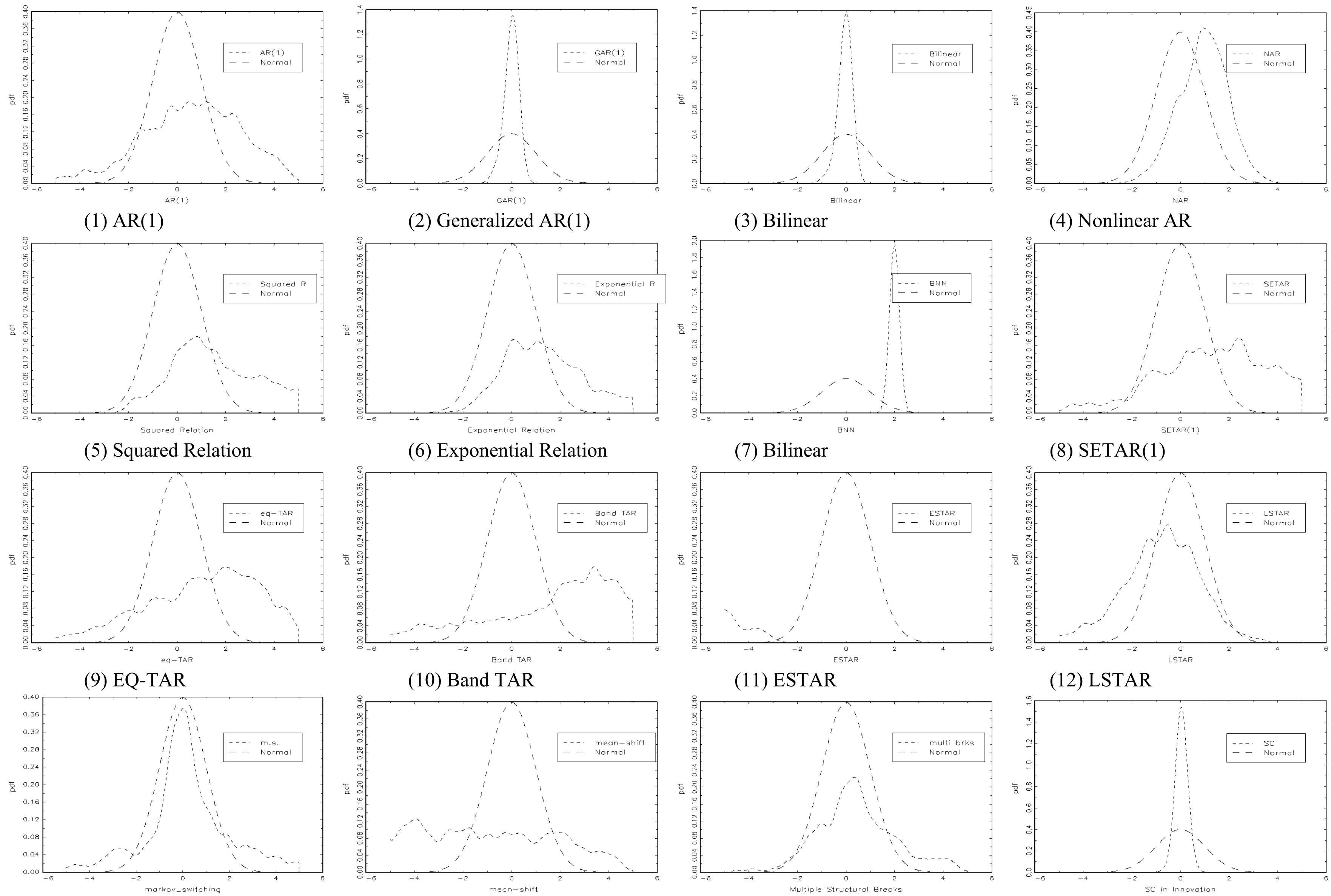
No Time Trend

Country	RALS(2&3)	$\hat{\rho}^2$	5% cv	ADF
Belgium	−2.967*	0.90	−2.810	−2.642
Canada	−2.167	0.92	−2.817	−2.013
Finland	−3.161*	0.78	−2.745	−2.240
France	−3.303*	0.76	−2.740	−3.296*
Italy	−4.544*	0.77	−2.741	−1.943
Japan	−6.346*	0.81	−2.758	−2.430
Luxembourg	−2.331	0.80	−2.752	−2.482
Netherlands	−3.140	0.59	−2.637	−3.201*
Norway	−3.359*	0.77	−2.745	−3.261*
Spain	−3.527*	0.83	−2.772	−2.311
UK	−3.746*	0.80	−2.754	−2.305
USA	−2.326	0.82	−2.764	−2.330

With Linear Trend

Belgium	−3.160	0.91	−3.338	−2.838
Canada	−2.324	0.92	−3.348	−2.189
Finland	−3.099	0.78	−3.246	−2.578
France	−3.328*	0.76	−3.235	−3.255
Italy	−4.424*	0.77	−3.244	−2.100
Japan	−6.329*	0.83	−3.287	−3.350
Luxembourg	−2.365	0.80	−3.268	−2.599
Netherlands	−3.867*	0.59	−3.079	−3.466*
Norway	−3.540*	0.77	−3.245	−3.594*
Spain	−3.774*	0.83	−3.290	−2.582
UK	−3.974*	0.80	−3.266	−2.528
USA	−2.302	0.82	−3.282	−2.401

*significant at 5%.



(1) AR(1) (2) Generalized AR(1) (3) Bilinear (4) Nonlinear AR

(5) Squared Relation (6) Exponential Relation (7) Bilinear (8) SETAR(1)

(9) EQ-TAR (10) Band TAR (11) ESTAR (12) LSTAR

(13) Markov-switching (14) Level Shift (15) Multiple Shift (16) Variance Shift

Figure 1. The Distribution of the Data Following Various Forms of Nonlinear Model